AP Calculus BC
Summer Review Packet

This packet is a review of information you learned in your previous math courses and are needed to be successful in AP Calculus BC.
The solutions to this packet are due no later than on the fourth school day in the fall. Your answers may be turned in on the first, second, third or fourth day of school. No packets turned in after the fourth day of school will be accepted under ANY circumstances other than with a doctor's note. This packet will count towards your first quarter grade.

Write out the ENTIRE solution to each problem including all steps. Answers alone will earn you ZERO credit.

There is a formula sheet at the end of this packet.

Complex Fractions

Simplify each of the following.

1. $\dfrac{\dfrac{25}{a}-a}{5+a}$

2. $\dfrac{2-\dfrac{4}{x+2}}{5+\dfrac{10}{x+2}}$

3. $\dfrac{4-\dfrac{12}{2x-3}}{5+\dfrac{15}{2x-3}}$

4. $\dfrac{\dfrac{x}{x+1}-\dfrac{1}{x}}{\dfrac{x}{x+1}+\dfrac{1}{x}}$

5. $\dfrac{1-\dfrac{2x}{3x-4}}{x+\dfrac{32}{3x-4}}$

Functions

Let $f(x)=2x+1$ *and* $g(x)=2x^2-1$**. Find each.**

6. $f(2)=$ ____________

7. $g(-3)=$ ____________

8. $f(t+1)=$ ____________

9. $f\left[g(-2)\right]=$ ____________

10. $g\left[f(m+2)\right]=$ ____________

11. $\dfrac{f(x+h)-f(x)}{h}=$ ______

Let $f(x)=\sin x$ **Find each exactly.**

12. $f\left(\dfrac{\pi}{2}\right)=$ ____________

13. $f\left(\dfrac{2\pi}{3}\right)=$ ____________

Let $f(x)=x^2,\ g(x)=2x+5,$ *and* $h(x)=x^2-1$**. Find each.**

14. $h\left[f(-2)\right]=$ ________

15. $f\left[g(x-1)\right]=$ ________

16. $g\left[h(x^3)\right]=$ ________

Find $\dfrac{f(x+h)-f(x)}{h}$ **for the given function *f*.**

17. $f(x)=9x+3$

18. $f(x)=5-2x$

Intercepts and Points of Intersection

Find the x and y intercepts for each.

19. $y=2x-5$

20. $y=x^2+x-2$

21. $y=x\sqrt{16-x^2}$

22. $y^2=x^3-4x$

Find the point(s) of intersection of the graphs for the given equations.

23. $\begin{aligned} x+y&=8 \\ 4x-y&=7 \end{aligned}$

24. $\begin{aligned} x^2+y&=6 \\ x+y&=4 \end{aligned}$

25. $\begin{aligned} x^2-4y^2-20x-64y-172&=0 \\ 16x^2+4y^2-320x+64y+1600&=0 \end{aligned}$

Interval Notation

26. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \le 4$		
	$[-1,7)$	
		

Solve each equation. State your answer in BOTH interval notation and graphically.

27. $2x-1\geq 0$

28. $-4\leq 2x-3<4$

29. $\frac{x}{2}-\frac{x}{3}>5$

Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

30. $f(x)=x^2-5$

31. $f(x)=-\sqrt{x+3}$

32. $f(x)=3\sin x$

33. $f(x)=\frac{2}{x-1}$

Inverses

Find the inverse for each function.

34. $f(x)=2x+1$

35. $f(x)=\frac{x^2}{3}$

Prove *f and g* are inverses of each other.

36. $f(x)=\frac{x^3}{2}$ $\quad g(x)=\sqrt[3]{2x}$

37. $f(x)=9-x^2,\ x\geq 0$ $\quad g(x)=\sqrt{9-x}$

Equation of a line

Slope intercept form: $y=mx+b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y-y_1=m(x-x_1)$

Horizontal line: y = c (slope is 0)

38. Use slope-intercept form to find the equation of the line having a slope of 3 and a y-intercept of 5.

39. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

40. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

41. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

42. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

43. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

44. Find the equation of a line passing through the points (-3, 6) and (1, 2).

45. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

Radian and Degree Measure

46. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

47. Convert to radians: a. 45° b. -17° c. 237°

Angles in Standard Position

48. Sketch the angle in standard position.

a. $\frac{11\pi}{6}$ b. 230° c. $-\frac{5\pi}{3}$ d. 1.8 radians

Reference Triangles

49. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

a. $\frac{2}{3}\pi$ b. 225°

c. $-\frac{\pi}{4}$

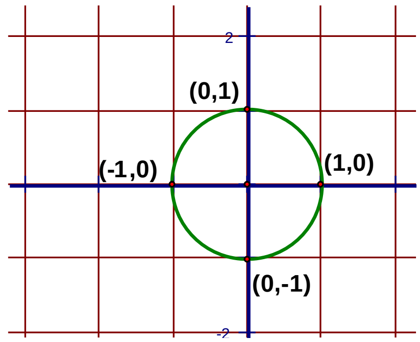
d. 30°

Unit Circle

50. $a.)\ \sin 180^{\circ}$ $b.)\ \cos 270^{\circ}$

$c.)\ \sin(-90^{\circ})$ $d.)\ \sin \pi$

$e.)\ \cos 360^{\circ}$ $f.)\ \cos(-\pi)$



Graphing Trig Functions

Graph two complete periods of the function.

51. $f(x) = 5\sin x$

52. $f(x) = \sin 2x$

53. $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

54. $f(x) = \cos x - 3$

Trigonometric Equations:

55. $\sin x = -\frac{1}{2}$

56. $2\cos x = \sqrt{3}$

57. $\cos 2x = \frac{1}{\sqrt{2}}$

58. $\sin^2 x = \frac{1}{2}$

59. $\sin 2x = -\frac{\sqrt{3}}{2}$

60. $2\cos^2 x - 1 - \cos x = 0$

61. $4\cos^2 x - 3 = 0$

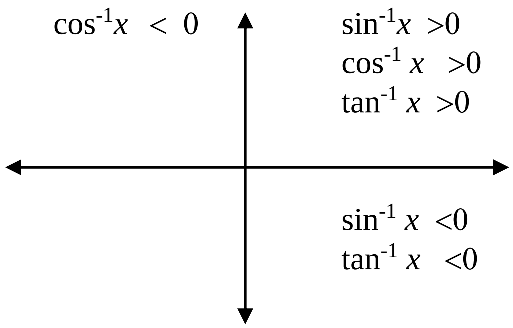
62. $\sin^2 x + \cos 2x - \cos x = 0$

Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of ways:

$$\arcsin(x) \qquad\qquad \sin^{-1}(x)$$

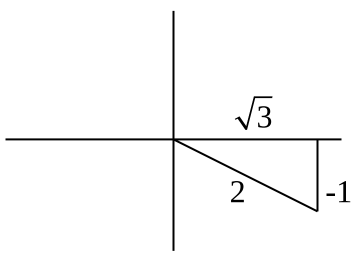
Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:

Express the value of "y" in radians.

$y = \arctan\frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, y = $-\frac{\pi}{6}$ so that it falls in the interval from

$$\frac{-\pi}{2} < y < \frac{\pi}{2} \qquad\qquad \text{Answer: } y = -\frac{\pi}{6}$$

For each of the following, express the value for "y" in radians.

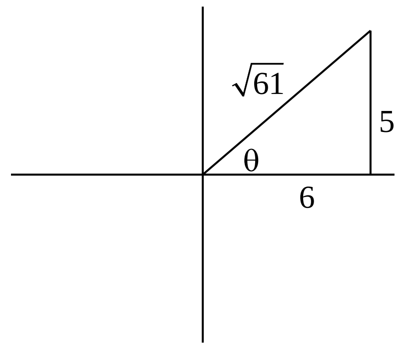
76. $y = \arcsin\frac{-\sqrt{3}}{2}$

77. $y = \arccos(-1)$

78. $y = \arctan(-1)$

Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$



Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.

$$\cos\theta = \frac{6}{\sqrt{61}}$$

For each of the following give the value without a calculator.

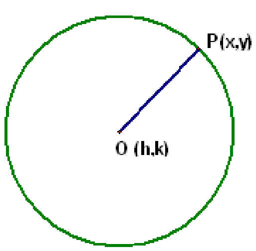
63. $\tan\left(\arccos\frac{2}{3}\right)$

64. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

65. $\sin\left(\arctan\frac{12}{5}\right)$

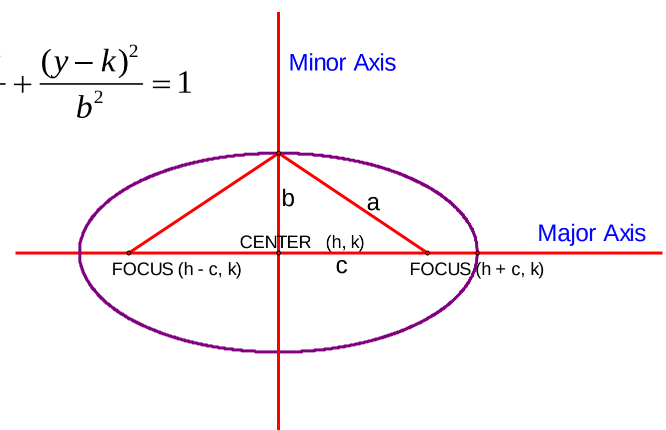
66. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

Circles and Ellipses



$$r^2 = (x-h)^2 + (y-k)^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

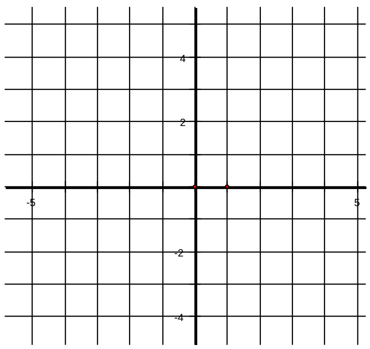


For a circle centered at the origin, the equation is $x^2 + y^2 = r^2$, where **r** is the radius of the circle.

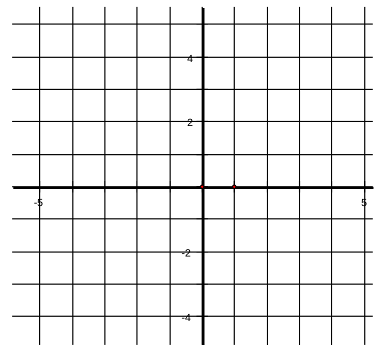
For an ellipse centered at the origin, the equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where **a** is the distance from the center to the ellipse along the x-axis and **b** is the distance from the center to the ellipse along the y-axis. If the larger number is under the y^2 term, the ellipse is elongated along the *y*-axis. For our purposes in Calculus, you will not need to locate the foci.

Graph the circles and ellipses below:

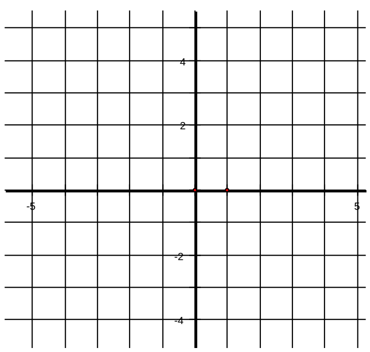
67. $x^2+y^2=16$



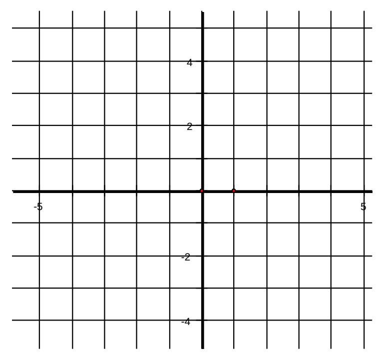
68. $x^2+y^2=5$



69. $\frac{x^2}{1}+\frac{y^2}{9}=1$



70. $\frac{x^2}{16}+\frac{y^2}{4}=1$



<u>Evaluating Limits Analytically</u>

Solve by direct substitution whenever possible. If needed, rearrange the expression so that you can do direct substitution.

76. $\lim_{x\to 2}(4x^2+3)$

77. $\lim_{x\to 1}\frac{x^2+x+2}{x+1}$

78. $\lim_{x\to 0}\sqrt{x^2+4}$

79. $\lim_{x\to \pi}\cos x$

80. $\lim_{x\to 1}\left(\frac{x^2-1}{x-1}\right)$ HINT: Factor and simplify.

81. $\lim_{x\to -3}\frac{x^2+x-6}{x+3}$

82. $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$ HINT: Rationalize the numerator.

83. $\lim_{x\to 3}\frac{3-x}{x^2-9}$

84. $\lim_{h\to 0}\frac{2(x+h)-2x}{h}$

Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

85. $f(x)=\frac{1}{x^2}$

86. $f(x)=\frac{x^2}{x^2-4}$

87. $f(x)=\frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

88. $f(x)=\frac{x^2-2x+1}{x^3+x-7}$

89. $f(x)=\frac{5x^3-2x^2+8}{4x-3x^3+5}$

90. $f(x)=\frac{4x^5}{x^2-7}$

Formula Sheet

Reciprocal Identities: $\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Quotient Identities: $\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \dfrac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p\log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim\limits_{h\to\infty} \dfrac{f(x+h)-f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: Ax + By + C = 0